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ELECTRIC AND MAGNETIC FIELDS

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16. Abstract

The general objective of this research is a fundamental physical understanding of electric and magnetic fields which, in turn, might promote the development of new concepts in electric space propulsion. The pproach taken is to investigate quantum representations of these fields. The objective and approach were not fully achieved during the support period covered by this report, but are included here to indicate some of the motivation for the direction taken in the research.

It is not possible, of course, to predict the exact line of basic research that will give rise to new propulsion concepts. However, some general observations can be made as to the line of research that might be expected to be more fruitful. To be consistent with our present most fundamental understanding of physical objects and their interactions, alternative representations should be consistent with quantum mechanics.

There is an existing quantum mechanical theory of electromagnetic interactions. A quantum representation of a static electric field is not considered within the context of quantum electrodynamics and the classical electromagnetic field theory upon which it builds. In exploring the nature of this exclusion, it is quickly found that the fundamental problem is one of long standing, but one that has only been partially understood.

In this study a number of energy-momentum anomalies have been described that result from the use of Abraham-Lorentz electromagnetic theory. These anomalies have in common the motion of charged bodies or current carrying conductors relative to the observer.

The anomalies can be avoided by using the nonflow approach, based on internal energy of the electromagnetic field. The anomalies can also be avoided by using the flow approach, if all contributions to flow work are included.

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FOREWORD

The general objective of this research is a fundamental physical understanding of electric and magnetic fields which, in turn, might promote the development of new concepts in electric space propulsion. The approach taken is to investigate quantum representations of these fields. The objective and approach were not fully achieved during the support period covered by this report, but are included here to indicate some of the motivation for the direction taken in the research.

The classical representations of electric and magnetic fields have been adequate for calculations related to conventional electric thrusters. In seeking fundamental departures from present electric propulsion concepts, though, the conventional representations may be much less useful.

It is not possible, of course, to predict the exact line of basic research that will give rise to new propulsion concepts. However, some general observations can be made as to the line of research that might be expected to be more fruitful. To be consistent with our present most fundamental understanding of physical objects and their interactions, alternative representations should be consistent with quantum mechanics.

There is an existing quantum mechanical theory of electromagnetic interactions. The proposed departure from the existing interpretations can perhaps be made clear by analogy with classical field theory. One approach used in classical electromagnetic theory is to sum the interactions of a particular charged particle with all other charged bodies in the system. An alternative approach, however, tends to bypass much of this complexity by emphasizing parameters of the electric and magnetic fields themselves. In this second approach the interaction of a

charged particle with electric and magnetic fields can be treated as one problem, while the system of particles giving rise to the electric and magnetic fields can be treated as a separate problem.

As a simple example of this difference in approach, consider the force on a charged particle between the plates of a parallel-plate capacitor. In the more lengthy particle interaction approach, the forces between the particle of interest and all other charged particles must be summed. Alternatively, the concept of an electric field can be used, and that field can be calculated in a simple manner by Gauss' law. The field then can be used to calculate the force on the charged particle.

Textbooks on electromagnetic theory are replete with other examples of the utility of the field concept, as opposed to working only with the charged particles themselves. In turning from classical electrodynamics to quantum electrodynamics, except for radiation, no equivalent quantum representation is found of electric and magnetic fields.

A quantum representation of a static electric field is not considered within the context of quantum electrodynamics and the classical electromagnetic field theory upon which it builds. In exploring the nature of this exclusion, it is quickly found that the fundamental problem is one of long standing, but one that has only been partially understood. The initial objective in this research, then, is to work toward a quantum representation of a static electromagnetic field.

TOWARD A QUANTUM MECHANICAL REPRESENTATION OF A

STATIC ELECTRIC FIELD

INTRODUCTION

The best present understanding of electromagnetic field interactions is a body of knowledge collectively known as quantum electrodynamics. In quantum electrodynamics, fields are generally treated using the Coulomb gauge, which is limited to radiation phenomena. In other words, the quantization of a static field is specifically excluded by the present techniques of field quantization.

The difficulties involved in quantizing electromagnetic fields without rectriction to radiation, though, involve more than the selection of a gauge.* The bulk of electromagnetic theory in use today is due to Abraham and Lorentz and can correctly be called Abraham-Lorentz electrodynamics.[1-6] The use of Abraham-Lorentz electrodynamics results, in the general case, in energy-momentum discrepancies. These discrepancies have been studied many times, with emphasis usually on the energy-momentum relations for an electron.[7-10]

If the electric field energy of an electron in the rest frame is U_o, special relativity requires that the associated equivalent mass be U_o/c^2. Observed from a reference frame in which the electron is moving at a velocity v, a momentum of $\gamma U_o v/c^2$ would clearly be expected. When the momentum of the electron is evaluated employing the techniques of

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*The electrodynamic formulations of many problems are underspecified without additional assumptions to remove possible ambiguity. The required further assumptions can be made by the selection of a "gauge".

Abraham-Lorentz electrodynamics, a momentum of $(4/3)(\gamma U_0 v/c^2)$ is obtained instead.

There is general agreement that the stress in what might be termed the body of the electron is central to the discrepancy.[7-10] This is the stress that Poincaré found necessary to hold the electron together,[11] and is often associated with his name. There is also general agreement that the expected momentum of $\gamma U_0 v/c^2$ can be obtained from electromagnetic theory if this stress, among other things, is properly included. From a formal mathematical viewpoint, using a covariant formulation, the problem can be considered solved.[6]

Two major aspects of the problem, though, would benefit from further explanation. One is the physical significance of the pressure-volume product for the assumed electron body, that has been involved in the momentum correction. The other is a more complete description of the shortcoming in Abraham-Lorentz electrodynamics that gives rise to this discrepancy. It should be emphasized that outmoded classical electron models are not a concern of this paper. The real concern is the interpretation of the energy-momentum discrepancy, which exists in both microscopic and macroscopic problems.

In the work presented herein, the covariant formulation of electromagnetic momentum will be reviewed and rationalized, from a physical viewpoint, with classical (Abraham-Lorentz) electromagnetic theory. In this rationalization, an alternate formalism will be presented for energy and momentum calculations of an electromagnetic field. It is hoped that this alternate formalism will promote new insights for possible quantum representations of electric and magnetic fields.

RADIATION FIELD QUANTIZATION

For a continuous field, ψ, defined at every space-time point $x_\mu =$ (x,y,z,ict), $\mu = 1,2,3,4$, the Euler-Lagrange equation is of the form

$$\frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathscr{L}}{\partial(\partial\psi/\partial x_\mu)}\right] - \frac{\partial \mathscr{L}}{\partial\psi} = 0 \tag{1}$$

where $\mathscr{L}$ is the Lagrangian.

Introducing the Maxwell stress tensor $F_{\mu\nu}$ and defining the current density four vector

$$J_\mu = (J_i, ic\rho) \qquad i = 2,3,4 \tag{2}$$

where c is the velocity of light and ρ is the charge density, Maxwell's equations are obtained from Eq. (1) if $\mathscr{L}$ is defined as

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu}F_{\mu\nu} + \frac{1}{c} J_\mu A_\mu \ . \tag{3}$$

The vector potential A_μ is defined by the relation

$$\frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} = F_{\mu\nu} \ . \tag{4}$$

The Hamiltonian for the free field ($\rho = 0$) can be written in terms of the magnetic and electric field as

$$H_{EM} = \frac{1}{2} \int [(\vec{B})^2 + (\vec{E})^2] \, d\vec{x} \tag{5}$$

Classical Radiation Field

A Fourier expansion of the vector potential can be written as

$$\vec{A}(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \sum_{\alpha} [c_{\vec{k},\alpha}(t)\vec{\varepsilon}^{(\alpha)}e^{i\vec{k}\cdot\vec{x}} + c_{\vec{k},\alpha}(t)\vec{\varepsilon}^{(\alpha)}e^{-i\vec{k}\cdot\vec{x}}] \tag{6}$$

where $\vec{\varepsilon}^{(\alpha)}$ is the polarization vector, $\vec{k}$ is the propagation momentum vector, and the c's are expansion coefficients.

Using the transversality condition

$$\vec{\nabla}\cdot\vec{A} = 0 \tag{7}$$

the Hamiltonian of the field can be written as

$$H_{EM} = \frac{1}{2} \int [|\vec{\nabla}\times\vec{A}|^2 + |\frac{1}{c}\frac{\partial\vec{A}}{\partial t}|^2] \, d\vec{x} \quad . \tag{8}$$

Assuming a time dependence

$$c_{\vec{k},\alpha}(t) = c_{\vec{k},\alpha}(0)e^{i\omega t} \tag{9}$$

where $\omega = ck$, and defining

$$Q_{\vec{k},\alpha} = \frac{1}{c} [c_{\vec{k},\alpha}(0) + c_{\vec{k},\alpha}^*(0)] \tag{10}$$

$$P_{\vec{k},\alpha} = \frac{-i\omega}{c} [c_{\vec{k},\alpha}(0) - c_{\vec{k},\alpha}^*(0)] \tag{11}$$

yields

$$H_{EM} = \sum_{\vec{k},\alpha} \frac{1}{2} [P^2_{\vec{k},\alpha} + \omega^2 Q^2_{\vec{k},\alpha}] \tag{12}$$

which is of the harmonic oscillator form.

Quantized Radiation Field

Defining operators

$$a_{\vec{k},\alpha} = \frac{1}{\sqrt{2\hbar\omega}} [\omega Q_{\vec{k},\alpha} + iP_{\vec{k},\alpha}] \tag{13}$$

$$a^+_{\vec{k},\alpha} = \frac{1}{\sqrt{2\hbar\omega}} [\omega Q_{\vec{k},\alpha} - iP_{\vec{k},\alpha}] \tag{14}$$

and recognizing that P and Q are canonical momenta and coordinates, and hence satisfy canonical commutation rules, it follows that

$$[a_{\vec{k},\alpha}, a^+_{\vec{k'},\alpha'}] = \delta_{\vec{k},\vec{k'}} \delta_{\alpha,\alpha'} \tag{15}$$

$$[a_{\vec{k},\alpha}, a_{\vec{k'},\alpha'}] = [a^+_{\vec{k},\alpha}, a^+_{\vec{k'},\alpha'}] = 0 \tag{16}$$

and

$$H_{EM} = \sum_{\vec{k},\alpha} [N_{\vec{k},\alpha} + \frac{1}{2}] \hbar\omega \tag{17}$$

where

$$N_{\vec{k},\alpha} = a^+_{\vec{k},\alpha} a_{\vec{k}\alpha} . \tag{18}$$

Thus, the field Hamiltonian is

$$H_{EM} = \sum_{\vec{k},\alpha} h\omega(a^+a + \tfrac{1}{2})$$ (19)

and the commutation rules for the a's are Bose-Einstein $[a_i, a_j^+] = \delta_{ij}$, etc. Thus the electromagnetic field is represented by a set of independent (non-interacting) bosons (i.e., photons).

In the case of a static field, a difference is surely that the Fourier expansion coefficients of the vector potential do not have an $e^{\pm i\omega t}$ time dependence, and that there is no requirement that $\omega = ck$, which is necessary to satisfy the wave equation in the traveling wave case.

ELECTROMAGNETIC ENERGY AND MOMENTUM

Previous Treatment of an Electron

The energy-momentum problems of Abraham-Lorentz electromagnetic theory have been studied primarily in the context of an electron.[7-10] (Wilson[8] assumes a charged sphere, but the problem is otherwise identical.) The treatments use various mathematical formalisms, but the underlying concepts are similar. The discussion presented here is intended to do justice to the substance of previous treatments, without becoming overly detailed.

A rigid, massless sphere is assumed for the body of an electron, with the charge distributed uniformly on the surface. Outside of this body, the radial electric field for an isolated, stationary electron, E_o, is

$$E_o = e/4\pi\epsilon_o r^2 \quad , \tag{20}$$

where e is the electronic charge, ϵ_o is the permittivity of free space, and r is the radius from the center of the electron (SI units). The internal energy of this field is obtained by integration,

$$U_o = \int \frac{1}{2} \epsilon_o E_o^2 d\tau_o \quad , \tag{21}$$

with the integration performed over the volume, τ_o, of the electric field, from the radius of the electron, r_e, to infinity, yielding

$$U_o = e^2/8\pi\epsilon_o r_e \ . \tag{22}$$

If the mass of the electron is viewed as being entirely electro-magnetic, the internal energy of the field has an equivalent rest mass of

$$m_e = U_o/c^2 \ . \tag{23}$$

When an isolated electron is observed with a steady relative velocity of v, the field energy must have an observed mass of

$$m = \gamma m_e = \gamma U_o/c^2 \tag{24}$$

and a momentum of

$$\vec{p} = \gamma m_e \vec{v} = \gamma U_o \vec{v}/c^2 \ , \tag{25}$$

where γ is $1/(1-v^2/c^2)^{1/2}$.

The preceding Eqs. (23) through (25) are quite simple, but, at the same time, fundamental in their importance. If the energy and momentum of a body do not satisfy Eq. (25), they cannot conform to the laws of physics as presently accepted.

Consider the momentum of the electron as calculated using the methods of Abraham-Lorentz electromagnetic theory. Starting with the electric field of the stationary electron, Lorentz transformations can be used to obtain the volume distributions of electric and magnetic fields for the el... oving at uniform velocity. The momentum of the electromagnetic fi... $\vec{p}_f$, is then obtained by the integration

$$\vec{P}_f = \int (\vec{E} \times \vec{H}) d\tau / c^2 , \tag{26}$$

where E and H are the transformed fields for the moving electron and τ is the corresponding volume. This integration gives the value

$$\vec{P}_f = (4/3)\gamma U_o \vec{v}/c^2 . \tag{27}$$

The field momentum p_f clearly differs from the expectation of Eq. (25), with the difference constituting the energy-momentum discrepancy for an electron obtained by Abraham-Lorentz electromagnetic theory.

The product of the pressure and volume for the body of the electron has the units of energy. The mass equivalent of this quantity of energy has a momentum that can be included in the total momentum of the system.

The pressure within the body of the electron is

$$P_b = -\frac{1}{2} \epsilon_o E_b^2 , \tag{28}$$

where E_b is the electric field at the surface of the electron body. This pressure can be calculated from the mutual repulsion of the electric charge distributed over the surface of election body. It is also the internal pressure Poincaré found necessary to balance the stress of the electric field.[11] Also note that the result is negative. The value of E_b can be obtained from Eq. (20), with the radius set equal to the radius of the electron body, r_e. With this substitution, the pressure is

$$P_b = -e^2/32\pi^2 \epsilon_o r_e^4 , \tag{29}$$

so that the product of pressure and volume is

$$P_b V_b = -e^2/24\pi\epsilon_o r_e \ . \tag{30}$$

From Eqs. (3) and (11), it is clear that

$$P_b V_b = -U_o/3 \ . \tag{31}$$

The momentum associated with this quantity of energy is

$$\vec{p}_b = -\gamma U_o \vec{v}/3c^2 \ . \tag{32}$$

When the momentum of the field, p_f, is added to this momentum for the electron body, p_b, the total momentum is

$$\vec{p} = \gamma U_o \vec{v}/c^2 \ , \tag{33}$$

which is in agreement with Eq. (25).

The calculation described constitutes the substance of previous treatments of this problem.[7-10] Although a massless electron body was assumed, a negative momentum was obtained for this body through the inclusion of a pressure-volume product in the momentum expression.

Macroscopic Charged Sphere

Further study of the energy-momentum discrepancy for an electron, along the lines described above, is of limited utility. Such a procedure would involve an outmoded model of an electron. Also, the discrepancy

can be shown to exist at the macroscopic level without reference to properties of fundamental particles. For the corresponding macroscopic problem, a large conducting sphere is assumed. This sphere would be expected to have some mass when neutral, but attention can be focused only on the additional energy and mass associated with a net charge.

To assure that the field energy has an equivalent mass far larger than the mass of the charging particles, it is assumed that

$$U_o/c^2 \gg m , \tag{34}$$

where m is the mass of the particles. The field energy can be obtained from Eq. (22), with the radius of the electron body, r_e, replaced by the radius of the sphere, r_s, and with the electronic charge, e, replaced by the net charge, q.

$$U_o = q^2/8\pi\epsilon_o r_s . \tag{35}$$

It should be noted that the inequality (34) can be met by making q sufficiently large, inasmuch as U_o increases as q^2.

To assure that the stress within the sphere remains below the elastic limit of the sphere material, that stress should be small. From Eq. (29), with substitutions similar to those for Eq. (35), the stress on the sphere is

$$P_s = -q^2/32\pi^2\epsilon_o r_s^4 . \tag{36}$$

Because this stress, an isotropic tension or negative pressure, varies

as r_s^{-4}, the stress can be reduced to any arbitrary value by making r_s sufficiently large.

The macroscopic regime can thus be assured by assuming a large enough net charge on the surface of a large sphere.

The Abraham-Lorentz momentum associated with the electromagnetic field of the sphere when it is in uniform translational motion is, from Eqs. (26) and (27),

$$\vec{p} = (4/3)\gamma U_o \vec{v}/c^2 , \tag{37}$$

with U_o the energy of the electric field when observed with the sphere at rest.

Returning to the matter of the stress within the sphere and examining it more carefully, the only energy within the sphere that results from the charging is the elastic deformation energy. The deformation energy per unit volume is, in general,

$$u_d = \frac{1}{2} \sigma^2/M , \tag{38}$$

where σ is the stress and M is the appropriate modulus of elasticity. The fractional deformation, or displacement, is σ/M. For a relatively rigid material,

$$\sigma/M \ll 1 . \tag{39}$$

The deformation energy of this rigid material per unit volume is, from Eq. (38),

$$u_d << \sigma .$$
<div style="text-align: right">(40)</div>

Energy transfer under the application of a stress requires a simultaneous displacement. This matter of displacement under stress application is critical. If an energy exists within a body as the result of a stres⁻ being applied to that body, then, from the conservation of energy, that energy must cross the boundary of the body during the application of the stress. For an energy to cross this boundary, the boundary must be displaced by the application of stress. If the displacement is negligible, so is the energy transfer.

The stress within the sphere is isotropic, and it equals the pressure, P. From Eq. (40), the deformation energy per unit volume of a rigid material under a pressure P must be much less than P. Mathematically, it would be possible to "correct" the momentum by including the pressure-volume product of the sphere in the energy-momentum calculation. But such an ap⁻oach cannot be justified from a viewpoint that requires energy to be conserved. By the selection of a reasonably rigid material for the sphere, the deformation energy within the sphere becomes negligible. For an isolated sphere, there is no other internal energy to include. Equation (37), then, gives the additional momentum due to charging the sphere. This result is clearly inconsistent with relativity.

Another aspect of this charged sphere problem should also be pointed out. The Poynting vector, $\vec{E} \times \vec{H}$, is associated with energy flow. For configurations in which charged bodies and current carrying conductors are at rest relative to the observer, the Poynting vector appears to correspond exactly to the actual energy flow.[12-13] But for a moving charged body, the situation is quite different.

Consider a field element associated with a moving charged sphere. The electric field is everywhere radial from the center of the sphere. (This is also true for the relativistic case.) If $\vec{E} \times \vec{H}$ is everywhere normal to $\vec{E}$, as well as varying in intensity with the angle of $\vec{E}$ relative to $\vec{v}$, it is apparent that $\vec{E} \times \vec{H}$ alone does not correspond to a real energy flow because such a flow would violate continuity.

Parallel-Plate Capacitor

A simple parallel-plate capacitor has not received the intensive study from the energy-momentum viewpoint that the electron has. This configuration, however, permits each component of electric field to be evaluated separately, thereby facilitating a more detailed examination of each component.

Parallel-plate capacitors are assumed to be oriented with the electric field directions parallel and normal to the velocity, as indicated in Figs. 1 and 2. The shaded portions represent insulators required to hold the plates apart against electrostatic attraction.

Each plate has a square projected area of dimension w on each side. The two plates are spaced a distance d apart. In addition to the parallel-field volume between the plates, there are fringe-field effects near the edges of the plates and small electric fields in other regions. All other electric field energy can be made negligible compared to that of the parallel-field volume by making w sufficiently large compared to d.

The pertinent Lorentz transformations are:

$$\ell_{\parallel} = \ell_{o\parallel}/\gamma , \qquad \ell_{\perp} = \ell_{o\perp} , \tag{41}$$
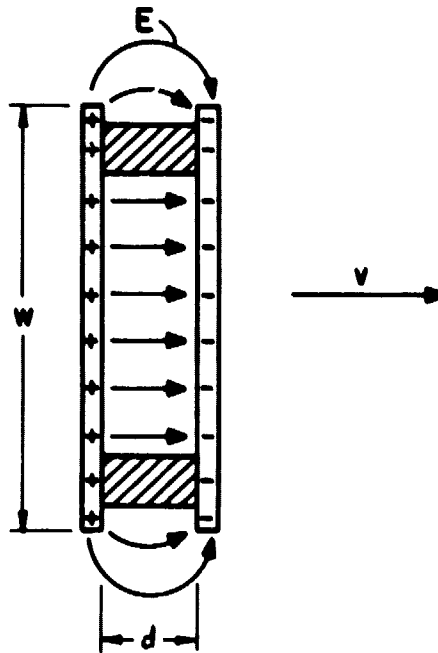
Fig. 1. Parallel-plate capacitor with field $\vec{E}$ parallel to velocity $\vec{v}$.
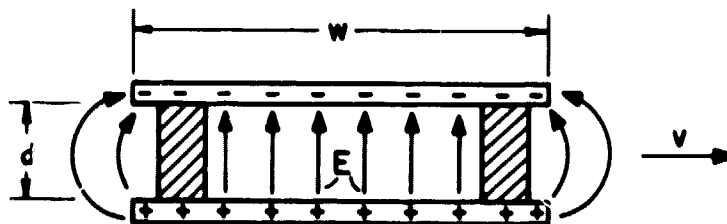

Fig. 2. Parallel-plate capacitor with field $\vec{E}$ normal to velocity $\vec{v}$.

$$\vec{E}_\| = \vec{E}_{o\|} \qquad \vec{E}_\perp = \gamma(\vec{E}_{o\perp} - \vec{v} \times \vec{B}_o) \ , \tag{42}$$

$$\vec{B}_\| = \vec{B}_{o\|} \qquad \vec{B}_\perp = \gamma(\vec{B}_{o\perp} + \vec{v} \times \vec{E}_o/c^2) \ . \tag{43}$$

The subscript o denotes the rest frame of the capacitor, and the sub-scripts $\|$ and $\perp$ refer to the electric field orientation with respect to $\vec{v}$. The velocity $\vec{v}$ is the velocity of the capacitor with respect to a fixed, or laboratory, inertial frame (i.e., $\vec{v}$ is measured in the fixed frame, not in the rest frame of the capacitor).

The field energy for the rest condition is given by Eq. (21). For a uniform electric field, the integration reduces to the product of energy density, $\frac{1}{2}\epsilon_o E_o^2$, and volume, τ_o.

$$U_o = \frac{1}{2}\epsilon_o E_o^2 \tau_o \ . \tag{44}$$

This means that, from special relativity, a momentum of

$$\vec{p} = \frac{1}{2}\gamma\epsilon_o E_o^2 \tau_o \vec{v}/c^2 \tag{45}$$

is expected from this field energy when the capacitor has relative velocity $\vec{v}$.

The momentum indicated by Abraham-Lorentz electromagnetic theory can also be calculated. For the configuration of Fig. 1, with the electric field parallel to $\vec{v}$,

$$\vec{B}_o = 0 \ , \tag{46}$$

$$\vec{E}_{o\perp} = 0 \ , \tag{47}$$

and, from Eqs. (42) and (43)

$$\vec{E}_{\parallel} = \vec{E}_{o\parallel} \qquad (48)$$

$$\vec{E}_{\perp} = \vec{B} = 0 . \qquad (49)$$

From Eq. (26), then,

$$\vec{p} = 0 . \qquad (50)$$

That is, no momentum is associated with the field energy due to the capacitor motion.

For the configuration of Fig. 2, with the electric field normal to $\vec{v}$,

$$\vec{B}_o = 0 , \qquad (51)$$

$$\vec{E}_{o\parallel} = 0 , \qquad (52)$$

and, from Eqs. (42) and (43),

$$\vec{E}_{\parallel} = \vec{B}_{\parallel} = 0 \qquad (53)$$

$$\vec{E}_{\perp} = \gamma \vec{E}_o , \qquad (54)$$

$$\vec{B}_{\perp} = \gamma \vec{v} \times \vec{E}_o / c^2 . \qquad (55)$$

Substituting the values for $\vec{E}$ and $\vec{B}$ into $\vec{E} \times \vec{H}$ yields

$$\vec{E} \times \vec{H} = \gamma^2 \epsilon_o E_o^2 \vec{v} . \tag{56}$$

From Eq. (41)

$$\tau = \tau_o / \gamma . \tag{57}$$

From Eqs. (26), (56), and (57), then,

$$\vec{p} = \gamma \epsilon_o E_o^2 \tau_o \vec{v} / c^2 . \tag{58}$$

This momentum is twice the expected value given by Eq. (45).

Having found the result for parallel and normal orientations of an electric field, a brief comparison can be made with the effect on a field from a charged sphere. Due to spherical symmetry, the rest-frame field energy of the spherical field can be represented as being divided equally between three mutually orthogonal directions. One of these three directions is taken to be parallel to $\vec{v}$, so that 1/3 of the field energy should have no momentum, in agreement with Eq. (51). The other two mutually orthogonal directions are both normal to $\vec{v}$, so that 2/3 of the field energy should have twice the expected momentum, in agreement with Eq. (56). For the total momentum, then, with only 2/3 of the energy having twice the expected momentum, the momentum is 4/3 of the expected value.

The field energy can also be considered for the two capacitor orientations. For the orientation of Fig. 1, the field is unaffected by the motion ($\vec{E}_{\parallel} = \vec{E}_{o\parallel}$) and Eq. (57) can be used for the volume change. The electromagnetic energy of the moving capacitor is thus

$$U = \frac{1}{2} \epsilon_c E_o{}^2 \tau_o/\gamma = U_o/\gamma . \qquad (59)$$

For the orientation of Fig. 2, the electric and magnetic fields are given by Eqs. (54) and (55), while the volume is again given by Eq. (57). The electromagnetic energy for the moving capacitor can be calculated from

$$U = \int (\frac{1}{2} \epsilon_o E^2 + \frac{1}{2} \mu_o H^2) d\tau , \qquad (60)$$

which gives

$$U = \frac{1}{2} \gamma \epsilon_o E_o{}^2 (1 + v^2/c^2) d\tau_o = \gamma U_o (1 + v^2/c^2) . \qquad (61)$$

It is found, then, that the electromagnetic energy is different for the two capacitor orientations, and that neither agrees with the expected value from relativity of γU_o. This result has been known for a long time. Trouton and Noble based an experiment to detect motion relative to the ether on this energy difference;[*] such an energy difference implies a torque on a suspended body under appropriate conditions.[14] A null result was obtained, which is consistent with fundamental concepts of relativity.

The experiment of Trouton and Noble was analyzed by Butler,[15] who concluded that the proper energy expression for a moving charged body is, in SI units,

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[*] Trouton and Noble did not consider the relativistic change in volume, Eq. (38), but an energy difference is predicted even when this effect is omitted.

$$U = \gamma^2 \int (\tfrac{1}{2} \epsilon_o E^2 - \tfrac{1}{2} \mu_o H^2) d\tau \qquad (62)$$

instead of Eq. (60). (See also p. 795 in Ref. 6.) It can be verified that Eq. (62) does, indeed, give an energy in agreement with relativity - when the initial conditions specify for the rest frame an electic field only. If a magnetic field only is specified for the rest frame, the magnitude of energy given by Eq. (62) is correct, but the sign is opposite.

Flow and Nonflow Processes

As an aid to the physical interpretations to follow, it is useful to describe thermodynamic flow and nonflow processes.

A nonflow process is concerned with internal energy. The internal energy per unit mass is obtained by integrating the specific heat at constant volume, C_v, from zero temperature to the temperature of interest.

$$u = \int_0^T C_v dT \qquad (63)$$

An example of a nonflow process is measuring a quantity of heat by heating a gas within a fixed volume container. This quantity of heat can be evaluated from the mass of the gas and the change in internal energy per unit of mass,

$$Q = m(u_2 - u_1) , \qquad (64)$$

where the subscript 2 indicates the final condition and 1 indicates the initial. The energy evaluation boundary in this example is the wall

of the container. In a nonflow process, no gas (or in the general case, fluid) crosses the energy evaluation boundary. The quantity of heat is thus directly equated to a change in internal energy within the gas. That is, to the various excitations and motions of gas molecules.

A flow process is concerned with enthalpy. The enthalpy per unit mass is obtained by integrating the specific heat at constant pressure, C_p,

$$h = \int_o^T C_p dT \qquad (65)$$

The enthalpy can also be expressed in terms of internal energy and the product of pressure, P, and volume per unit mass, V.

$$h = u + PV \qquad (66)$$

The product PV in Eq. (66) is called flow work. An example of a flow process is measuring a heating rate by heating a gas that is flowing through a pipe at constant pressure. This heating rate can be calculated from the mass flow rate and the enthalpy change per unit of mass.

$$\dot{Q} = \dot{m}(h_2 - h_1) = \dot{m}[u_2 - u_1 + P(V_2 - V_1)] \qquad (67)$$

Note that only part of the heating rate corresponds to a change in internal energy, u. The remainder corresponds to a change in the flow work, PV, and represents an energy exchange that takes place between the gas and the surroundings. To be more specific, a flow process evaluation is normally concerned with a fluid flowing past a boundary. The pressure-volume product of that fluid thus represents an energy

transport across the boundary, in addition to the internal energy of the fluid.

The concepts of flow and nonflow processes have been included here in a very abbreviated manner. More detailed descriptions are available for these two processes from a thermodynamic point of view.[16-17] A treatment that includes relativistic effects in the two processes has apparently not been pursued in standard thermodynamics texts.

Rest Frame for Electromagnetic Energy

For the steady translational velocities of interest herein, the electromagnetic energy of an isolated charged body or current carrying conductor can be regarded as remaining associated with the body or conductor. The rest frame for the body or conductor is thus also the rest frame for that electromagnetic field energy.

For a charged body, with only an electric field observed in the rest frame, the rest energy is, of course, given by Eq. (21). For a current carrying conductor, with only a magnetic field observed in the rest frame, the rest energy is given in a similar manner by

$$U_o = \int \frac{1}{2} \mu_o H_o^2 d\tau_o , \tag{68}$$

with the integration performed over the volume, τ_o, of the magnetic field.

For a system involving both charged bodies and current carrying conductors, both electric and magnetic fields can be present in the same volume when observed from the rest frame. The Poynting vector could be used for a detailed description of the energy flow. Further, the local

values of $\vec{E}_o$ and $\vec{H}_o$ could be used to define a direction and velocity for this energy flow, at each point in the electromagnetic field.

It is of interest, however, to be able to calculate the overall electromagnetic energy in the rest frame when both electric and magnetic fields are present in the same volume. This energy is:

$$U_o = \int \left(\frac{1}{2}\,\varepsilon_o E_o^{\,2} + \frac{1}{2}\,\mu_o H_o^{\,2}\right) d\tau_o \; . \tag{69}$$

In using this expression, it is helpful to recognize that

$$\int (\vec{E} \times \vec{H}) d\tau = 0 \; . \tag{70}$$

That is, that the net energy flow must be zero in the rest frame.

Flow and Nonflow Electromagnetic Formalisms

It is customary in thermodynamics to use energy per unit of fluid mass. For an electromagnetic field, an energy per unit of volume is more appropriate. In this section, the general approach followed is similar to that presented in a short earlier study.[18-19]

From the preceding discussion of the rest frame for electromagnetic energy, the internal energy density in that frame is defined as

$$u_o = \frac{1}{2}\,\varepsilon_o E_o^{\,2} + \frac{1}{2}\,\mu_o H_o^{\,2} \; . \tag{71}$$

For the enthalpy, the flow energy must be added to the internal energy. The flow work per unit volume is simply the stress. And for

the nonisotropic stress of an electromagnetic field, the stress
associated with energy transfer is the stress in the direction of
motion, $\sigma_{o\|}$. The enthalpy is, for $v \ll c$,

$$h_o = u_o + \sigma_{o\|} . \tag{72}$$

The stress, in terms of electric and magnetic field components parallel
and normal to the translation velocity, is

$$\sigma_o = \frac{1}{2} \epsilon_o (E_{o\perp}^2 - E_{o\|}^2) + \frac{1}{2} \mu_o (H_{o\perp}^2 - H_{o\|}^2) \tag{73}$$

Substituting the internal energy of Eq. (71) and the stress of Eq. (73)
into Eq. (72), yields, again for $v \ll c$,

$$h_o = \epsilon_o E_{o\perp}^2 + \mu_o H_{o\perp}^2 . \tag{74}$$

Thus only the transverse electric and magnetic field components
contribute to enthalpy.

Using the nonflow approach, the internal energy density of Eq.
(71) can be integrated to obtain the total internal energy

$$U_o = \int u_o d\tau_o \tag{75}$$

For an exclusively electrostatic field in the rest frame, Eq. (75) is
equivalent to Eq. (21). With the broader definition of internal energy
given by Eq. (71), Eq. (75) is clearly a generalization of Eq. (21). If
it is assumed that the charged bodies and current carrying conductors

are rigid, the internal energy is limited to the volume of the electromagnetic field, and the integration can also be limited to that volume.

A flow process evaluation normally involves a boundary, with the fluid (in this case an electromagnetic field) flowing past this boundary. Because the configurations of interest do not change with time when viewed from the rest frame, a spatial integration can be substituted for the temporal integration that might otherwise be expected. For a flow process evaluation, then, the total electromagnetic energy should be

$$U_o = \int h_o d\tau_o . \tag{76}$$

Strictly speaking, this evaluation should be for a near-rest frame, rather than the rest frame. This is because the flow work depends on the stress in the direction of motion. The field stress is nonisotropic, and a velocity is necessary to establish flow direction, even though $v \ll c$. Using Eq. (72), Eq. (76) can be divided into two integrals.

$$U_o = \int u_o d\tau_o + \int \sigma_{oll} d\tau_o \tag{77}$$

In using U_o as the sum of these two integrals, it is implied that the second integral is zero (see Eq. (75)). This implication follows directly from the conditions for stationary equilibrium. A system of charged bodies, current carrying conductors and connecting members is assumed to be in equilibrium when viewed from the rest frame. Being in equilibrium, all forces in all directions sum to zero over any plane passing through the system. Treating the volume integral as the summation over successive planes, this summation must also equal zero.

Note that, although both the flow and nonflow evaluations give the same total electromagnetic energy, the flow process integration must include the volume of any stressed members in the system. These members may be rigid, so that the actual internal energy associated with the stress is negligible, but the flow evaluation still requires the inclusion of flow work contributions from them.

For a translation velocity that is significant compared to the velocity of light, the energy and volume transformations require that

$$u = \gamma^2 u_o , \tag{78}$$

$$h = \gamma^2 h_o . \tag{79}$$

The corresponding expressions for energy flux, $\vec{S}$, are

$$\vec{S}_n = u\vec{v} , \tag{80}$$

$$\vec{S}_f = h\vec{v} , \tag{81}$$

where subscripts n and f refer to nonflow and flow evaluations. The expressions for momentum density, $\vec{g}$, are

$$\vec{g}_n = u\vec{v}/c^2 , \tag{82}$$

$$\vec{g}_f = h\vec{v}/c^2 . \tag{83}$$

Either u or h can be integrated spatially to verify that

$$U = \gamma U_o \, . \tag{84}$$

The same result can also be obtained by a temporal integration of energy flux, $\vec{S}$, through a boundary that is fixed relative to the observer. In either the spatial or temporal integration, the nonflow evaluation (depending on u and u_o) can be limited to the electromagnetic field volume. In the flow evaluation (depending on h and h_o), the integrations must include flow work contributions of various members due to stresses caused by the electromagnetic field.

For momentum, both the nonflow and flow momentum densities can be integrated to show that

$$\vec{p} = U\vec{v}/c^2 \, . \tag{85}$$

The same limits on the regions of integration for $\vec{p}$ apply as in the previous integrations of both U and U_o.

A note of caution should be included here against too detailed an interpretation of electromagnetic field properties when both electric and magnetic fields are present in the same volume, when observed in the rest frame. The definitions of u_o and h_o were selected to give the correct integrated values of U_o, U, and $\vec{p}$ when considering an entire closed system. In such a rest frame, however, there will in general be energy flow loops, defined by the Poynting vector. If a localized definition of rest frame is required, such that the condition of Eq. (70) is met locally, such a definition will in general vary from point to point throughout the electromagnetic field. If such a definition is required, the localized rest frame will in general have electric and magnetic fields either parallel or antiparallel. The definition of rest

frame used in this paper for an entire closed system loses some of this detailed field information, but gains in having only a single transformation velocity for the entire electromagnetic field.

No similar problem exists for either an exclusively electric field in the rest frame, or an exclusively magnetic field in the rest frame. In either case, there is no energy flow when viewed from the rest frame. A velocity transformation is therefore correct both for overall field integral values and point-by-point local values.

Interpretation of Discrepancies

It is of interest to describe, in as general terms as possible, the problems in which energy-momentum discrepancies are encountered in Abraham-Lorentz electrodynamics. These problems all have in common charged bodies and/or current carrying conductors that are moving relative to the observer. In what is herein defined as the rest frame, there are no such discrepancies.

When moving bodies are present, the discrepancies appears to be inherent in Abraham-Lorentz electrodynamics and, for the most part, can be explained in terms of flow and nonflow processes.

Most of the formalism of Abraham-Lorentz electrodynamics corresponds to the nonflow approach. As examples, the volume integration of $\frac{1}{2} \epsilon_o E_o^2$ to obtain a capacitor energy, or the volume integration of $\frac{1}{2} \mu_o H_o^2$ to obtain an inductor energy, are precisely equivalent to the nonflow integration of internal energy.

On the other hand, Poynting vector energy flow and classical momentum density correspond more to a flow process treatment. At least in the context of electrodynamics, the flow process must be viewed as

considerably less familiar to most workers. Returning to the charged

sphere problem, the pressure-volume product for the sphere must be

included in the Abraham-Lorentz momentum calculation. This is because

it is a flow work term and the integration of flow work must cover all

parts of the moving system, not just the electromagnetic field volume.

For the rest frame, the energy flow and momentum density are in

complete agreement for Abraham-Lorentz electrodynamics and the flow

process approach. When moving charged bodies or current carrying

conductors are involved, the agreement can be less complete. For a

moving charged sphere, for example, both the Poynting vector energy flow

and momentum density are everywhere normal to the radial electric

field (Fig. 3). As was pointed out, such a distribution of energy flow

cannot satisfy continuity. For the flow process approach, however, both

energy flow and momentum are everywhere parallel to the translation

velocity for the sphere. Examined in detail, these parameters for the

flow process will be found to correspond to the component parallel to $\vec{V}$

of the electrodynamic parameters. When integrated over a symmetrical

charge and electric field distribution, only the component parallel to

$\vec{V}$ remains. Hence the integrated values are in complete agreement with

the flow process evaluations.

The major shortcoming in some of the typical procedures used in

Abraham-Lorentz electrodynamics, then, is that two different energy

evaluation approaches are used in the same body of knowledge, usually

without a clear distinction as to the differences between the two

approaches. With an appreciation of the differences between flow and

nonflow processes, and with an understanding of where the two processes

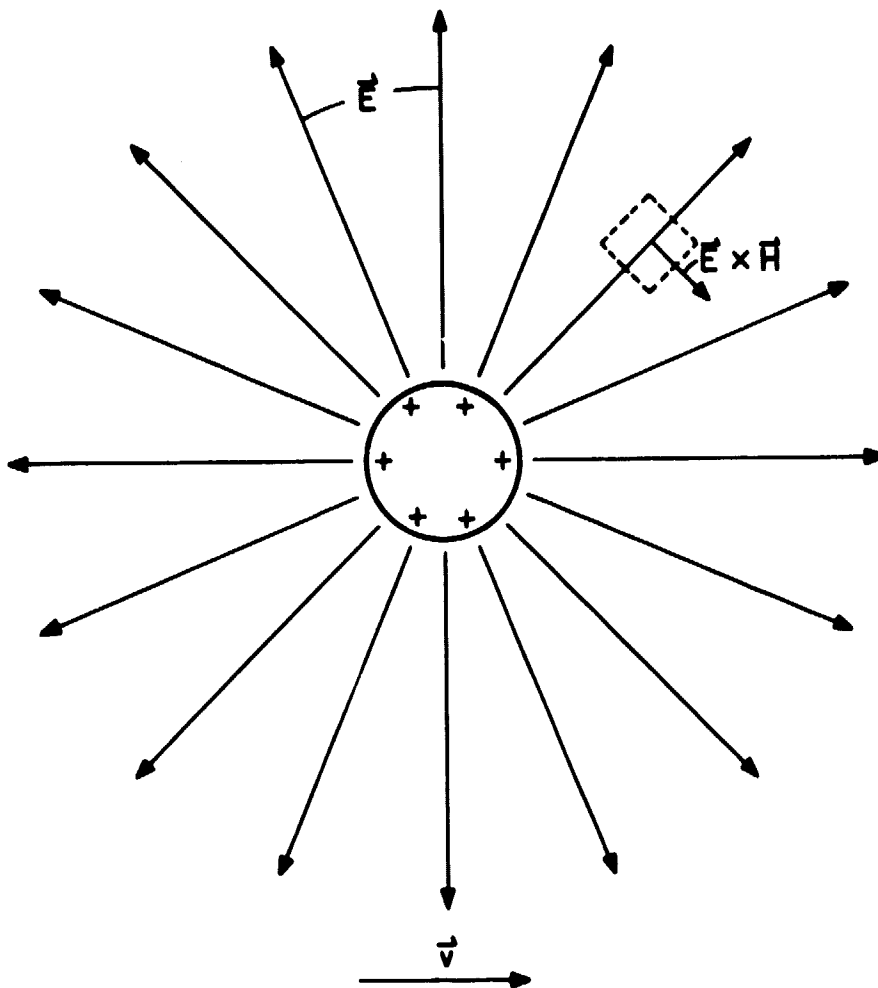are involved, the energy-momentum discrepancies are readily resolved.

Fig. 3. Electric field of moving charged sphere. Note Poynting vector direction for element of electric field. (Element of field shown by dashed line.)

TREATMENT OF STATIC FIELDS

Gauge Transformations

A certain amount of arbitrariness is injected into the solution of problems in classical electromagnetism when, for convenience, potentials are used instead of working with the fields directly. Because differential operators are employed to extract the fields from the potentials, the potentials are arbitrary to within additive functions that vanish when the operator is employed.

If vector and scalar potentials $\vec{A}$ and Φ can be employed to represent electric and magnetic fields $\vec{B}$ and $\vec{E}$ through the relationships:

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{86}$$

and

$$\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t} \quad , \tag{87}$$

then a general gauge transformation to new potentials $\vec{A}'$ and Φ' using

$$\vec{A}' = \vec{A} + \vec{\nabla}\Psi \tag{88}$$

and

$$\Phi' = \Phi - \frac{\partial \Psi}{\partial t} + \Phi_o \tag{89}$$

where Ψ is an arbitrary scalar field that leaves $\vec{B}$ and $\vec{E}$ unchanged.

The value of the divergence of the vector potential can be viewed as an adjustable parameter in gauge transformations. Thus, when

$$\vec{\nabla} \cdot \vec{A} = 0 \tag{90}$$

the gauge is said to be the Coulomb gauge, the radiation gauge, or the transverse gauge. All of the terms are synonymous. If

$$\vec{\nabla} \cdot \vec{A} = -\frac{1}{c}\frac{\partial \Phi}{\partial t} \tag{91}$$

the gauge is said to be the Lorentz gauge.

Gauge for Static Field

The approach initially taken in an attempt to quantize static electric and magnetic fields was to follow the procedure for field quantization developed by Sakurai.[20] To follow this procedure faithfully, it was necessary to choose a gauge in which the scalar potential vanished and the divergence of the vector potential also vanished. This can be stated as

$$\phi' = 0 \tag{92}$$

and

$$\vec{\nabla} \cdot \vec{A}' = 0 \ . \tag{93}$$

As a starting point, it was determined that scalar and vector potentials

$$\phi = -\vec{E}_o \cdot \vec{r} \tag{94}$$

and

$$\vec{A} = \frac{1}{2} \vec{B}_o \times \vec{r} \tag{95}$$

could be used to represent static electric and magnetic fields $\vec{E}_o$ and $\vec{B}_o$. However, this gauge manifestly did not meet the criteria in Eqs. (92) and (93). A scalar field

$$\psi = -\vec{E}_o \cdot \vec{r}t \tag{97}$$

is used to generate a gauge transformation to satisfy Eqs. (92) and (93) where

$$\vec{A}' = \frac{1}{2} \vec{B}_o \times \vec{r} - \vec{\nabla}(\vec{E}_o \cdot \vec{r}t) \tag{99}$$

and

$$\phi' = 0 . \tag{100}$$

So far, attempts to quantize both static electric and static magnetic fields using this gauge have not been successful.

CONCLUDING REMARKS

The approach taken in the quantization of classical electromagnetic fields has been outlined along with some observations important in attempts to quantize a static field.

In this study a number of energy-momentum anomalies have been described that result from the use of Abraham-Lorentz electromagnetic theory. These anomalies have in common the motion of charged bodies or current carrying conductors relative to the observer.

The anomalies can be avoided by using the nonflow approach, based on internal energy of the electromagnetic field. The anomalies can also be avoided by using the flow approach, if all contributions to flow work are included.

The Abraham-Lorentz approach for either energy flow or momentum density most closely approximates the flow process approach. The energy-momentum problem has been studied repeatedly, though, without apparent recognition that the stress-volume product of a moving charged body is a flow-work term, and not a real energy located within that body.

Further, a detailed examination of some aspects of Abraham-Lorentz electromagnetic theory has, in general, shown components of energy flow and momentum density normal to a general translational motion. These normal components cannot be reconciled with the flow theory approach, a detailed relativistic accounting, or continuity of energy flow. One should, therefore, conclude that some of the methods used in Abraham-Lorentz electromagnetic theory are not necessarily relativistically correct for energy and momentum evaluations of electromagnetic energy moving with charged bodies and/or current carrying conductors. Some of the difficulties can be overcome if it is understood that both flow and

nonflow evaluations are included without distinction between the two processes.

Either the flow or nonflow alternate formalisms should be better suited than Abraham-Lorentz theory to the quantization of static, or near-static, electromagnetic fields.

Attempts to cast the static problem into the form customarily used in field quantization yield time-dependent potentials and have not yet led to a successful resolution of the quantization problem.

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